Ch 5: Numerical Growth

5 latro to Series	• •		
Def. Series	• •		
Lei Juns _{nal} de la sequence. An infinite series is an expression of the form	• •	• •	
$\sum_{n=1}^{\infty} (a_1 + a_2 + a_3 + a_4 + \dots + 1) $ This is a tormal expression Since we don't know what			
internetional internetion and the second strategy			
$E_{x} \lesssim L_{-1} \times L_{x} $	• •	• •	• •
$\sum_{n=1}^{\infty} n^{-1} + 2 + 3 + 4 + \dots (\text{flarmonic Jerles})$			
$\sum_{n=1}^{\infty} \sin(\frac{n\pi}{2}) = \sin(0) + \sin(\frac{\pi}{2}) + \sin(\pi) + \dots = 0 + +0- +0+ +0- +\dots$			
$\sum_{n=0}^{n=0} (-n)^{n} = (-1)^{1} + (-2)^{2} + (-3)^{3} + (-4)^{9} + 1 + 4 - 27 + 256 + 1$	• •	• •	
$\sum_{n \in I} (n) = (1) + (2) + $	• •	• •	
Def. Partial Succession			
If Σ_{an} is a series, its sequence of partial sums, Σ_{a} is defined as			
$S_{\mu} = a_{\mu} + a_{\mu} + a_{\mu} $ (sum up to a_{K})	• •		• •
$F_{r} = F_{r} \sum_{k=1}^{r} h_{r}$	• •		
$S_1 = 1$, $S_2 = 3$, $S_3 = 6$, $S_4 = 10$	• •	• •	0 0
Def Companya of a Series			
A series $\sum_{n=1}^{\infty} a_n$ converges to sell if $\lim_{k \to \infty} S_k = S$ and we call S the sum of the series.			
IF {Sx} diverges, the series diverges.	• •	• •	• •
$\sum_{n=0}^{n} e_n(\frac{n}{2}) = 0 + +0- +0+ +0- +\cdots$			
has partial sums So=0, S1=1, Sz=1, S3=0, S4=0, S5=1, S6=1, S7=0, S5=0,	• •		
$k = \sum_{i \in \mathcal{I}} \sum_{i \in \mathcal{I}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}} \sum_{i \in \mathcal{I}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}} \sum_{i \in \mathcal{I}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}} \sum_{i \in \mathcal{I}}$	• •		
· [∞ 2 [*] (· <u>·</u> · <u>·</u>) = (- · <u>·</u>) + (<u>+</u> · <u>·</u>) + ((<u>+</u> · <u>·</u>) + (<u>·</u>) + (<u>+</u> · <u>·</u>) + (<u>·</u>) + (<u>+</u> · <u>·</u>) + (
$\pi_{1}(n_{1}+1)$ (1 2) (2 3) (3 4) (4 5)	• •		
has partial sums らこ にち, らzこにち, らgこにも So Sull = lot to	• •	• •	
$\sim < \kappa$, κ_1 Cinc lim $(1 - \frac{1}{2}) = 1$ the the ceries conjectives and $\sum_{i=1}^{\infty} (1 - \frac{1}{2}) = 1$ (Telescond S	enies		
Since $\lim_{k \to \infty} S_k = \lim_{k \to \infty} \left(1 - \frac{1}{k+1} \right) = 1$, then the series converges and $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right) = 1$ (Telescoping S	enies)	• •	
Since $\lim_{k \to \infty} S_k = \lim_{k \to \infty} (1 - \frac{1}{k+1}) = 1$, then the series converges and $\sum_{n=1}^{\infty} (\frac{1}{n} - \frac{1}{n+1}) = 1$ (Telescoping S The Harmonic Series $\sum_{k=1}^{\infty} diverges$	eries)	• •	
Since $\lim_{k \to \infty} S_k = \lim_{k \to \infty} (1 - \frac{1}{k+1}) = 1$, then the series converges and $\sum_{n=1}^{\infty} (\frac{1}{n} - \frac{1}{n+1}) = 1$ (Telescoping S The Harmonic Series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. Assume for a contradiction that the series converges to SGIR.	enies)	· · ·	• •
Since $\lim_{k \to \infty} S_k = \lim_{k \to \infty} (1 - \frac{1}{k+1}) = 1$, then the series converges and $\sum_{h=1}^{\infty} (\frac{1}{h} - \frac{1}{h+1}) = 1$ (Telescoping S The Harmonic Series $\sum_{k=1}^{\infty} \frac{1}{h}$ diverges. Assume for a contradiction that the series converges to SEIR. Then $S = (1 + \frac{1}{2}) + (\frac{1}{3} + \frac{1}{4}) + (\frac{1}{3} + \frac{1}{6}) + \dots$	enies)	· · · · · · · · · · · · · · · · · · ·	· · ·
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Since $\lim_{k \to \infty} S_k = \lim_{k \to \infty} (1 - \frac{1}{k+1}) = 1$, then the series converges and $\sum_{h=1}^{\infty} (\frac{1}{h} - \frac{1}{h+1}) = 1$ (Telescoping S The Harmonic Series $\sum_{k=1}^{\frac{N}{2}-1}$ diverges. Assume for a contradiction that the series converges to SEIR. Then $S = (1 + \frac{1}{2}) + (\frac{1}{3} + \frac{1}{4}) + (\frac{1}{3} + \frac{1}{6}) + (\frac{1}{3} + \frac{1}{6}) + \dots$ $> (\frac{1}{2} + \frac{1}{2}) + (\frac{1}{4} + \frac{1}{4}) + (\frac{1}{2} + \frac{1}{6}) + (\frac{1}{3} + \frac{1}{6}) + \dots$ $= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ = S	enies)	· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·
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Since $\lim_{k \to \infty} S_k = \lim_{k \to \infty} (1 - \frac{1}{k+1}) = 1$, then the series converges and $\sum_{n=1}^{\infty} (\frac{1}{n} - \frac{1}{n+1}) = 1$ (Telescoping S The Harmonic Series $\sum_{n=1}^{\infty} 1$ diverges. Assume for a contradiction that the series converges to SEIR. Then $S = (1 + \frac{1}{2}) + (\frac{1}{3} + \frac{1}{4}) + (\frac{1}{5} + \frac{1}{5}) + (\frac{1}{7} + \frac{1}{6}) + \dots$ $> (\frac{1}{2} + \frac{1}{2}) + (\frac{1}{4} + \frac{1}{4}) + (\frac{1}{5} + \frac{1}{5}) + (\frac{1}{7} + \frac{1}{6}) + \dots$ $= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ = 5 Hence $S > S$. Thus a contradiction, so the Harmonic Series diverges. Note: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{T^2}{6}$. .

D.C. Contractions	· · · · · · · · · · · · · · · · · · ·
A geometric series is a series of the form	· · · · · · · · · · · · · · · · · · ·
$\sum_{n=0}^{\infty} \Gamma^{n} = +\Gamma + \Gamma^{2} + \Gamma^{3} + \Gamma^{4} + \dots \text{ for some rGR}$	
Case 1. $r=1$ Case 2. $r=-1$	<u>Case 3. r≠±1</u>
$\sum_{n=0}^{\infty} \left[\prod_{i=1}^{n} + + + + +\dots \right]_{n=0}^{\infty} \left[-1 \right]^{n}$	$\int_{K^{-1}} \int_{K^{-1}} \int_{K^{-1}$
$S_k = k + 1$ and $k^{1.5}_{100} S_k = \infty$, $S_k = \{0, if k \in odd$ and $k^{1.5}_{100} S_k$. UNE, so the series diverges.	Thus $\lim_{k \to \infty} S_k = \lim_{k \to \infty} \frac{ -r^{k+1} }{ -r } \approx \frac{1}{ -r } \lim_{k \to \infty} 1 - r^{k+1} = \frac{1}{1-r}$
· · · · · · · · · · · · · · · · · · ·	exactly when Irl<1. The limit DNE otherwise
Thm 1. Geometric Series Test The geometric series 50,00 converges if Irl<1 and diverges other	Xwise.
f rl≤ , then \$° ~	
5.4 Arithmetic of Series	
Thm 3 Arithmetic for Series I Let 2 an = A and 2 br = Bin and CER Then	
$\int \sum_{n=1}^{\infty} ca_n = cA$	
$2\sum_{n=1}^{\infty}(a_n+b_n)=A+B$	
The 4' Arithmetic for Serie II	
I. If $\sum_{n=1}^{\infty}$ an converges, then $\sum_{n=1}^{\infty}$ an converges for each $j \ge 1$. No	te, $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \dots + a_{j-1} + \sum_{n=1}^{\infty} a_n$
2. If $\sum_{n=1}^{\infty}$ an converges for some j, then $\sum_{n=1}^{\infty}$ an converges.	finite sum
	anying time sums account artest annagence.
We can change finitely many terms and not affect convergence.	
5.2 Geometric Series (contid)	
$\sum_{n=0}^{\infty} \left(-\frac{1}{4}\right)^{n} = \frac{1}{1-\left(-\frac{1}{4}\right)} = \frac{4}{5} \text{since } \left -\frac{1}{4}\right < 1$	· · · · · · · · · · · · · · · · · · ·
$\frac{\text{Ex.2}}{\sum_{n=1}^{\infty} \left(\frac{1}{2^n}\right)} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} = \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{2} \left(\frac{1}{1-\frac{1}{2}}\right) = 1 \text{since} \left \frac{1}{2}\right $	<pre></pre>
$\sum_{n=0}^{\infty} \frac{4^{2n}}{5^{2n+1}} = \sum_{n=0}^{\infty} \frac{(4^{2n})^{n}}{(5^{2n})^{n}} = \frac{1}{5} \sum_{n=0}^{\infty} \left(\frac{16}{125}\right)^{n} = \frac{1}{5} \left(\frac{1}{1-\frac{16}{125}}\right) = \frac{25}{109} \text{since } \left \frac{16}{125}\right < 1$	
$ \sum_{n=1}^{9} 6 \cdot 25^n 4^{-2n-2} = \sum_{n=1}^{\infty} \frac{ 6 \cdot 25^n}{4^{2n} \cdot 4^2} = \sum_{n=1}^{\infty} \left(\frac{25}{16}\right)^n = \sum_{n=0}^{\infty} \left(\frac{25}{16}\right)^{n+1} = \frac{25}{16} \sum_{n=0}^{\infty} \left(\frac{25}{16}\right)^n $ diver	nges since <u>25 </u> > ⇒) original series diverges

55 Express 11.62 B as a fraction		• •		•	• •		• •		• •	•	•	• •	 •	•	• •	
$ 1.62\overline{13} = 1.62 + \frac{18}{10000} + \frac{18}{100000} + \frac{18}{1000000} + \frac{18}{1000000000}$	1000000000															
$=\frac{1167}{100}+\frac{16}{100}+\frac{18}{$	f														• •	
$= \frac{162}{100} + \frac{18}{104} \left(1 + \frac{1}{102} + \frac{1}{104} + \frac{1}{104} \right)$	· · · ·) · · ·	• •	• •	•	• •		• •		• •	•	•		•	•	• •	
$= \frac{1162}{100} + \frac{18}{100} \left(1 + \frac{1}{100} + \frac{1}{100}\right)$	• • • • • • • • • • • • •	• •	• •	•			• •	•	• •	•	•			•	• •	
$= \frac{1162}{100} + \frac{18}{5} \sum_{n=1}^{\infty} \left(\frac{1}{100}\right)^n$. .			•		•		•		•	•		 •	•	• •	•
$\frac{100}{100} = \frac{1162}{100} \pm \frac{18}{100} \left(\frac{1}{100} \right)$					• •				• •	0						
$\frac{100}{3196} + \frac{107}{107} \left(\frac{1 - \frac{1}{100}}{1 - \frac{1}{100}} \right)$		• •			• •	•	• •			•					• •	
275		• •	• •	•	• •	•	• •		• •	•	•			•	• •	

56.6 A laser beam is fired at a planet with an innershield and outershied. Each shield reflects to absorbs 2, and transmits to of the beam. If the beam's intensity is I, what fraction passes through both shields

		Τ.Ι.Ι	I I			• •				• •	
I I/8	I/82 = 7/44	<u><u></u> <u> <u> </u> <u></u></u></u>	† 163 .	64 *	• • •	• •					
1/4.8		$=\frac{1}{1}\left(1+\frac{1}{1}\right)+\frac{1}{1}$		- + .	1	• •	•	•			
		· · · 64 · (· · · · 16 · · · 16	· · · · · · · · · · · ·	•	J	•					
I/4-32 I/42.8	- · · · · ·	<u></u> <u>Γ</u> ⁵ ⁵ (⊥)"	• •	• •		• •					
	L/4282 = 1/16-64	64	• •	• •		• •				• •	
I/43.8		$I = \frac{I}{I} \left(\frac{I}{I} \right) I = I$	• •	• •		• •			•	• •	
		$641 (-9_{16})$		• •		• •			•	• •	
		. <u>_ I (6</u> · · · ·				• •					
<u>.</u>	<u>I/44.82 = I/162.64</u>	. 64 [•] IS									
		= I since III	1								
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5.3 Divergence Test

We can focus on determining if a series converges or diverges generally without finding the sum. The If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n = 5$ for some SER. Proof Let $\{S_n\}$ be the sequence of partial sums, so $S_n = a_1 + a_2 + ... + a_n + ... + a_n$, $S_{n-1} = a_1 + a_2 + ... + a_{1n} + ... + a_{n-1}$. Note: $\lim_{n \to \infty} S_n = S = \lim_{n \to \infty} S_{n-1}$. Since $a_n = S_n - S_{n-1}$, then $\lim_{n \to \infty} a_n = \lim_{n \to \infty} S_n - S_{n-1} = S - S = 0$ Then 2. Divergence Test If $\lim_{n \to \infty} a_n \neq 0$ or $\lim_{n \to \infty} a_n$ DNE, then $\sum_{n=1}^{\infty} a_n$ diverges. Contropositive: If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \to \infty} a_n = 0$. Note: Div test can be used to spot a divergent series before using a complicated test, so try it I^{st} .

E.1 $\sum_{n=1}^{\infty} \frac{n}{n+10^{10}}$ Since $\lim_{n \to \infty} \frac{n}{n+10^{10}} = 1 \neq 0$, the series diverges by Div test.	• •	•	• •	•
$E_{x,2} \sum_{n=1}^{\infty} (-1)^n$ Since $\lim_{n \to \infty} (-1)^n$ DNE, the series diverges by Div test	• •	•	• •	•
E.3 $\sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$ Since $\lim_{n \to \infty} \left(\frac{4}{3}\right)^n = \infty$, the series diverges by Div test	• •	•	• •	•
$\frac{E_{x}4}{n} \sum_{n=1}^{\infty} \frac{\sin(n)}{n}$ Since $\lim_{n \to \infty} \frac{\sin(n)}{n} = 0$, so Div test is inconclusive		•	••••	•
5.5 & 5.6 Tests for Positive Series			• •	•
Def. Positive Series A series Zan is positive if an≥O ∀n∈N		•	• •	•
- The Integral Test		•		
Thm 8. Integral lest Let f be a function that is 1. continuous on $[0,\infty)$ 2. positive on $[1,\infty)$ 3. decreasing on $[1,\infty)$ and let $a_n = f(n)$. Then $\sum_{n=1}^{\infty} a_n$ converges $\iff \int_{1}^{\infty} f(x) dx$ converges	[m,00 care)		
$\frac{\frac{1}{2}}{\frac{1}{2}} \frac{1}{2} $	· · ·		· · ·	
(\Leftarrow) Suppose $\sqrt{f}(d)$ converges.	• •	•	• •	•
Since $\int_{k=2}^{\infty} \int_{k=2}^{\infty} \int_{k=2}^{\infty} \int_{k=2}^{\infty} \int_{k=2}^{\infty} a_{k} \leq a_{1} + \int_{k=2}^{\infty} \int_{k=2$	• •	•	• •	•
(⇒) Suppose $\int_{k=1}^{\infty} f(z) dz diverges from above, so \lim_{n \to \infty} \int_{k=1}^{\infty} f(z) dz = \inftyWe know \int_{k=1}^{\infty} f(z) dz \leq \sum_{k=1}^{\infty} a_n = S_{n-1}, so \lim_{n \to \infty} S_{n+1} = \infty$	• •	•	· ·	•
~ ≥ an diverges ■		•	• •	•

$\mathbb{E}_{\mathbf{k}} \left(\sum_{j=1}^{\infty} \frac{1}{n^2} \right)$		· ·	• •	• •		• •	· ·	
Consider $f(\infty) = \frac{1}{2}c_2$, which is cts, pos, and decr on [],	, ⁰⁰) 50	Hhe	Integra	Test	applies		• •	• •
$\int_{1} \frac{1}{2^{2}} dx \text{ converges } (p-\text{test } p=2>1), \text{ so } \sum_{n=1}^{\infty} \frac{1}{n^{2}} \text{ converges }$	5		• •	• •			• •	
5.2 5 $\ln(h)$								
「「」」」) lim ho(h) 4HR lim 古一〇 co 川」 D: Tath Gile	• •		• •					
$\int_{1}^{1} \frac{1}{100} = \int_{1}^{1} \frac{1}{100} $	0 0			• •			• •	
For decr. $f(x) = \frac{1-hx}{x} \le 0 \Rightarrow -hx \le 0 \Rightarrow \le hx \Rightarrow x \ge e$.	• •	· ·	0 0	• •		• •	• •	
$\int_{\infty}^{\infty} dx = \lim_{h \to \infty} \left(\frac{h(x)}{h(x)} dx = \lim_{h \to \infty} \frac{(h(x))^2}{(h(x))^2} \right ^b = \infty, \text{ so } \sum_{h \to \infty}^{\infty} \frac{h(x)}{h(x)} dx$	liverges.							
	• •		• •	0 0			0 0	
The grip Series Test	1			• •				
The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges to p>1 and diverges to p>1.		• •		•••			• •	• •
$E_{x} \sum_{n=1}^{\infty} \frac{1}{n^{3}n}$ converges (p-series, p= $\frac{3}{2} > 1$)							• •	
$\sum_{i=1}^{\infty} \frac{1}{2} = \text{diverges} (p - \text{series}, p = \frac{1}{2} \le 1)$	0 0		0 0	0 0			0 0	
		• •				• •	• •	
		• •	• •	• •		• •	• •	
Note that we can't use the Integral Test to compute the	. sum o	fac	onverge	nt ser	ies i	• •	· ·	• •
Note that we can't use the Integral Test to compute the For example, $\int \frac{1}{2} \frac{1}{2} dx = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{7L^2}{6}$ But we can approximate sums, with the Integral Test	. sum o	Fac	onverge	nt ser	ies	· · ·	· · ·	• •
Note that we can't use the Integral Test to compute the For example, $\int_{\frac{1}{2}}^{\frac{1}{2}} dx = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{7L^2}{6}$ But we can approximate sums with the Integral Test.	. Sum of	Fac	onverge	nt ser	ies	· · ·	· · ·	• •
Note that we can't use the Integral Test to compute the For example, $\int \frac{1}{2^2} dx = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{7L^2}{6}$ But we can approximate sums with the Integral Test. Def. Remainder For a convergent series $\sum_{n=1}^{\infty} a_n = SER$, the remainder is the	. sum of	- a c	converge using S	nt ser	ies ipproxima	e S is		
Note that we can't use the Integral Test to compute the For example, $\int \frac{1}{2^2} dx = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{7L^2}{6}$ But we can approximate sums with the Integral Test. Def. Remainder For a convergent series $\sum_{n=1}^{\infty} a_n = SEIR$, the remainder is the the error when using Sn to approximate S is given by $B_n = S = S_n = 0$ and $t = 0$. sum of	when	converge using S	nt ser	ies Approxima	e S is		
Note that we can't use the Integral Test to compute the For example, $\int_{-\frac{1}{2}}^{\frac{1}{2}} dx = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{7L^2}{6}$ But we can approximate sums with the Integral Test. Def. Remainder For a convergent series $\sum_{n=1}^{\infty} a_n = SEIR$, the remainder is the the error when using Sn to approximate S is given by $R_n = S - S_n = a_{n+1} + a_{n+2} + \dots$	error	when	converge using S	nt ser in to c	ies Ipproxima	e S is		
Note that we can't use the Integral Test to compute the For example, $\int \frac{1}{2} dx = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{7L^2}{6}$ But we can approximate sums with the Integral Test. Def. Remainder For a convergent series $\sum_{n=1}^{\infty} a_n = SER$, the remainder is the the error when using Sn to approximate S is given by $R_n = S - S_n = a_{n+1} + a_{n+2} +$ Thus 8. Integral Test and Estimation of Sums and Errors If $a_n = f(r)$ when $f(r)$ is the remainder on [1 (20) then	error	when	converge using S	nt ser	ies Ipproxima	eSi≘		
Note that we can't use the Integral Test to compute the For example, $\int \frac{1}{2}z dx = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{7L^2}{6}$ But we can approximate sums with the Integral Test. Def. Remainder For a convergent series $\sum_{n=1}^{\infty} a_n = SER$, the remainder is the the error when using Sn to approximate S is given by $R_n = S - S_n = a_{n+1} + a_{n+2} +$ Thus 8. Integral Test and Estimation of Sums and Errors If $a_n = f(x)$ where $f(x)$ is ct_{n-1} pos, and decr on $[1, \infty)$, then $\int_{1}^{\infty} f(c) dx \leq R_n \leq \int_{1}^{\infty} f(c) dx (from the proof of)$	error	ac	converge using S	nt ser	ies ipprovina	e S is		
Note that we can't use the Integral Test to compute the For example, $\int \frac{1}{2\pi} dx = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{7L^2}{6}$ But we can approximate sums with the Integral Test. Def. Remainder For a convergent series $\sum_{n=1}^{\infty} a_n = SER$, the remainder is the the error when using Sn to approximate S is given by $R_n = S - S_n = a_{n+1} + a_{n+2} + \dots$ Thus 8. Integral Test and Estimation of Sums and Errors If $a_n = f(x)$ where $f(x)$ is ct_{n-1} pos, and decr on $[1, \infty)$, then $\int_{n=1}^{\infty} f(x) dx \leq R_n \leq \int_{n=1}^{\infty} f(x) dx$ (from the proof of) We get the upper bound on the remainder.	error	when	converge using S	nt ser in to c	ies ipprovina	e S is		
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Note that we can't use the Integral Test to compute the For example, $\int \frac{1}{x^2} dx = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{7L^2}{6}$ But we can approximate sums with the Integral Test. Def. Remainder For a convergent series $\sum_{n=1}^{\infty} a_n = SER$, the remainder is the the error when using Sn to approximate S is given by $R_n = S - S_n = a_{n+1} + a_{n+2} + \dots$ Thus 8. Integral Test and Estimation of Sums and Errors If $a_n = f(x)$ where $f(x)$ is ct_n , po_n , and decr on $[1, \infty)$, then $\int_{n=1}^{\infty} f(cx) dx \leq R_n \leq \int_{n=1}^{\infty} f(cx) dc$ (from the proof of) $s_n = S_n = a_n + \int_{n=1}^{\infty} f(cx) dc$ (from the proof of) We get the upper bound on the remainder. We can improve our estimate, rather than just use S_n , $S_n + \int_{n=1}^{\infty} f(cx) dx \leq S \leq S_n + \int_{n=1}^{\infty} f(cx) dx$	error	when	using S	nt ser n to c		e S is		
Note that we can't use the Integral Test to compute the For example, $\int \frac{1}{32} dx = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{7L^2}{6}$ But we can approximate sums with the Integral Test. Def. Remainder For a convergent series $\sum_{n=1}^{\infty} a_n = SER$, the remainder is the the error when using Sn to approximate S is given by $R_n = S - S_n = a_{n+1} + a_{n+2} +$ Thus 8. Integral Test and Estimation of Sums and Errors If $a_n = f(x)$ where $f(x)$ is ct_n pos, and decr on $[1, \infty)$, then $\int_{n=1}^{\infty} f(c) dx \leq R_n \leq \int_{n=1}^{\infty} f(c) dc$ (from the proof of) $S = S_n$ we get the upper bound on the remainder. We can improve our estimate, rather than just use Sn, $S_n + \int_{n=1}^{\infty} f(c) dx \leq S \leq S_n + \int_{n=1}^{\infty} f(c) dc$	error	vher -	using S	nt ser n to c		<mark>e S i</mark> ≘		
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Note that we can't use the Integral Test to compute the For example, $\int \frac{1}{\sqrt{2}} dx = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^n} = \frac{\pi}{6}^n$ But we can approximate sums with the Integral Test. Def. Remainder For a convergent series $\sum_{n=1}^{\infty} a_n = SEIR$, the remainder is the the error when using Sn to approximate S is given by $R_n = S - S_n = a_{n+1} + a_{n+2} + \dots$ Thus 8. Integral Test and Estimation of Sums and Errons If $a_n = f(x)$ where $f(x)$ is ct_n pos, and decr on $[1,\infty)$, then $\int_{n=1}^{\infty} f(c) dx \leq R_n \leq \int_{n=1}^{\infty} f(c) dc$ (from the proof of) the Integral Test We get the upper bound on the remainder. We can improve our estimate, rather than just use S_n , $S_n + \int_{n=1}^{\infty} f(c) dx \leq S \leq S_n + \int_{n=1}^{\infty} f(c) dc$ Take the midpoint to approximate S with the error at most half of the width of the interval.	error	ohen -	using S	nt ser		e S is		

Ex.1 Find an upper bound on the error if we use S_{10} to approximate $\sum_{n=1}^{\infty} \frac{1}{n^3}$ $R_{10} \leq \int_{0}^{\infty} \frac{1}{x^3} dx = \lim_{b \to \infty} \int_{0}^{\infty} \frac{1}{x^3} dx = \lim_{b \to \infty} -\frac{1}{2x^2} \Big _{10}^{b} = \lim_{b \to \infty} -\frac{1}{2b^2} - \left(-\frac{1}{2(10)^2}\right) = \frac{1}{200} = 0.005$ \Rightarrow the error is at most 0.005	· ·	•	•	· ·	•	•	• • •
E2. How many terms are needed to approximate $\sum_{n=1}^{\infty} \frac{1}{n^2}$ with an error of at most 0.00005 Rn $\leq \int_{n}^{\infty} \frac{1}{\pi^2} d\omega c = \frac{1}{2n^2}$ We need $\frac{1}{2n^2} \leq 0.0005$ $\Rightarrow 2n^2 \geq 2000$ $\Rightarrow n^2 \geq 1000$ $\Rightarrow n \geq 1000 \approx 31.23$ $\Rightarrow n \geq 32$ for the desired accuracy	· · · · · · · · · · · · · · · · · · ·	•	•	· · · · · · · · · · · · · · · · · · ·	•	•	• • • •
Ex3 Considering $\sum_{i=1}^{\infty} \frac{1}{n^3}$, we know using Side to approximate S $S_{10} + \int_{1}^{\infty} \frac{1}{x^3} dx \leq S \leq S_{10} + \int_{1}^{\infty} \frac{1}{x^3} dx$ $\Rightarrow 1.201664 \leq S \leq 1.202532$ * better than Ex.1 $\therefore S \approx 1.202098$ and the error is actually 0.0005 (round up)		•	•	· · ·	•	•	•
 The Comparison Test Thm 6. Comparison Test Assume O= an = bn for nEN (or eventually) 1. If Σ bn converges, then Σ an converges. 2. If Σ an diverges, then Σ bn diverges. Proof of (2): Let ISn3 and ISS¹³ be the sequence of portial sume for Σ an and Σ bn respectively. 	· · · · · · · · · · · · · · · · · · ·	-	•				• • • • • •
If $\sum_{n=1}^{\infty} a_n$ diverges, then since $a_n \ge 0$ \forall_n , $\lim_{n \to \infty} S_n^{n} = \infty$ But since $a_n \le b_n$ \forall_n , $S_n^n \le S_n^n$ \forall_n , we have $\lim_{n \to \infty} S_n^{b} = \infty$ \therefore b_n diverges \therefore (1) is the contrapositive of (2) and so is true. $f_{n-1} = \sum_{n=1}^{\infty} \frac{n^2}{n}$ $S_{n-1} = \frac{n^2}{n^2}$ $S_{n-1} = \frac{n^2}{n^2} = \frac{1}{n^2}$ $S_{n-1} = \frac{1}{n^2}$	μ 		• • • • • •	· · · · · · · · · · · · · · · · · · ·		•	• • • • •
the given series converges by comparison. Ex.3 $\sum_{n=1}^{\infty} \frac{n^2 + n}{n^4 - 8}$ For $n \ge 2$, $\frac{n^2 + n}{n^4 - 8} \ge \frac{n^2 + n}{n^4} \ge \frac{n^2}{n^4} = \frac{1}{n^2} \ge 0$ Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series $p=2>1$), the comparison test is inconclusive. But this series "looks like" $\sum_{n=1}^{\infty} \frac{1}{n^2}$, so we expect it to converge.			•	· · · · · · · · · · · · · · · · · · ·		•	

- The Limit Comparison Test	· · · · · · · · · · · · · · ·
Thm 7. Limit Comparison Test (LCT) If an≥O and bn>O for nEN (or eventually) and here bn = L then 1. If LE(0,∞), then Zan converges ⇔ Zbn converges 2. If L=O and Zbn converges, then Zon converges. If L=O and Zan diverges, then Zbn diverges. 3. If L=∞ and Zan converges, then Zbn converges. If L=∞ and Zan converges, then Zbn converges. If L=∞ and Zbn diverges, then Zbn diverges.	· ·
<u>Proof (1)</u> : Suppose $\lim_{n\to\infty} \frac{a_n}{b_n} = L \in (0,\infty)$. Then $\exists m, M \in (0,\infty)$ so that $m \leq \frac{a_n}{b_n} \leq M$ for n sufficient We get $mb_n \leq a_n \leq Mb_n$ for n sufficient large. By Companison Test Σa_n converge/diverge iff Σb_n converge/diverge.	Harge. M
$\frac{Proof}{O} = m \leq \frac{a_n}{b_n} \Rightarrow mb_n \leq a_n$	· · · · · · · · · · · · ·
$\mathbf{E}_{\mathbf{x}} \begin{bmatrix} \sum_{n=1}^{\infty} \frac{n^2 + n}{n^2 + n} \end{bmatrix} = \mathbf{E}_{\mathbf{x}} \begin{bmatrix} \sum_{n=1}^{\infty} \frac{n^2 + 1}{n^2 + n} \end{bmatrix}$	Ex.3 <u>5 In(n)</u>
Apply LCT with $\tilde{\Sigma}_{1}$ $\frac{4^{\circ}}{3^{\circ}}$	Use LCT with 💐 🕂
$\lim_{n \to \infty} \frac{\frac{n^2 - n}{2}}{\frac{1}{2}} = \lim_{n \to \infty} \frac{n^4 + n^3}{n^4 - 8} = e(0, \infty) \qquad \qquad \lim_{n \to \infty} \frac{\frac{4^{n+1}}{3^2 + n}}{\frac{4^n}{2}} = \lim_{n \to \infty} \frac{ 2^n (+ \frac{1}{4^n})}{ 2^n + n^{4n}} = e(0, 0) $	
Since Since (p-series, p=2>1) Since Since (qeo with r +3 >1),	Since Since diverges (Harmonic)
the given series converges by LCT the given series converges by LCT Note. Div test could work	the given series diverges.
the given series converges by LCT the given series converges by LCT Note. Div test could work	the given series diverges.
the given series converges by LCT the given series converges by LCT Note. Div test could work 5.7 Alternating Series	the given series diverges.
The given series converges by LCT The given series converges by LCT Note. Div test could work 5.7 Alternating Series Def: Alternating Series	the given series diverges.
the given series converges by LCT The given series converges by LCT Note. Div test could work 5.7 Alternating Series Def: Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form	the given series diverges.
the given series converges by LCT The given series converges by LCT Note. Div test could work 5.7 Alternating Series Def. Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{i=1}^{n+1} (-1)^{n+1} a_n = a_i - a_2 + a_3 - a_4 + \dots$	the given series diverges.
the given series converges by LCT the given series converges by LCT Note: Div test could work 5.7 Alternating Series Def: Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 + \dots$	the given series diverges.
the given series converges by LCT the given series converges by LCT Note. Div test could work 5.7 Alternating Series Def: Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 + \dots$ $\sum_{n=1}^{\infty} (-1)^n a_n = -a_1 + a_2 - a_3 + a_4 - \dots$	the given series diverges.
the given series converges by LCT The given series converges by LCT Note. Div test could work 5.7 Alternating Series Def. Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 + \dots$ $\sum_{n=1}^{\infty} (-1)^n a_n = -a_1 + a_2 - a_3 + a_4 - \dots$ provided that $a_n > 0 \ \forall n$.	the given series diverges.
The given series converges by LCT The given series converges by LCT Note: Div test could work 5.7 Alternating Series Def: Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 + \dots$ provided that $a_n > 0 \ \forall n$. Ex. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{11} + \dots$	the given series diverges.
The given series converges by LCT The given series converges by LCT Note: Div test could work 5.7 Alternating Series Def: Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 +$ $\sum_{n=1}^{\infty} (-1)^n a_n = -a_1 + a_2 - a_3 + a_4$ provided that $a_n > 0 \ \forall n$. Ex. $ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = -\frac{1}{2} + \frac{1}{3} - \frac{1}{4} +$	the given series diverges.
The given series converges by LCT the given series converges by LCT Note: Div test could work 5.7 Alternating Series Def: Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 + \dots$ $\sum_{n=1}^{\infty} (-1)^n a_n = -a_1 + a_2 - a_3 + a_4 - \dots$ provided that $a_n > 0$ Vn. Ex. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ Then 10: Alternating Series Test (AST)	He given series diverges.
the given series converges by LCT the given series converges by LCT Note: Div test could work 5.7 Alternating Series Def: Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 + \dots$ provided that $a_n > 0$ $\forall n$. Ex. $ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = -\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ Then 10: Alternating Series Test (AST) Let $a_n > 0$ $\forall n$ and consider the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ [f. $\int_{1}^{a_3} \frac{(-1)^{n+1}}{n} = (-1)^{n+1} a_n = a_1 + a_2 - a_3 + a_4 - \dots$	the given series diverges.
the given series converges by LCT The given series converges by LCT Note: Div test could work 5.7 Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 +$ $\sum_{n=1}^{\infty} (-1)^n a_n = -a_1 + a_2 - a_3 + a_4$ provided that $a_n > O$ $\forall n$. Ex. $\int_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} +$ Then 10: Alternating Series Test (AST) Let $a_n > O$ $\forall n$ and consider the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$. If $1 \cdot \{a_n\}$ is (eventually) decreasing: $a_n \ge a_{n+1}$	the given series diverges.
the given series converges by LCT the given series converges by LCT Note: Div test could work 5.7 Alternating Series Def. Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{n=1}^{\infty} (-1)^{n} a_n = a_1 + a_2 - a_3 + a_4 - \dots$ provided that $a_n > 0$ Vn. Ex. $\int_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ The IO. Alternating Series Test (AST) Let $a_n > 0$ Vn and consider the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$. If $1 \cdot \{a_n\}$ is (eventually) decreasing: $a_n \ge a_{n+1}$ 2. $\lim_{n \ge n} a_n = 0$ Then $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges.	He given series diverges.
the given series converges by LCT the given series converges by LCT Note: Div test could work 5.7 Alternating Series Def: Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 +$ provided that $a_n > 0$ Vn. Ex. $\int_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} +$ Then 10: Alternating Series Test (AST) Let $a_n > 0$ Vn and consider the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$. If 1 fands is (eventually) decreasing: $a_n \ge a_{n+1}$ 2. $\lim_{n \ge 1} a_n = 0$ Then $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges.	He given series diverges.
the given series converges by LCT the given series converges by LCT Note: Div test could work 5.7 Alternating Series A series is alternating if terms are alternating positive and negative. Series of the form $\sum_{n=1}^{\infty} (-1)^{n+1} = \alpha_1 - \alpha_2 + \alpha_3 - \alpha_4 + \dots$ provided that $a_n > 0$ $\forall n$. Ex. $\int_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ The 10: Alternating Series Test (AST) Let $a_n > 0$ $\forall n$ and consider the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1}a_n$. If 1. fand is (eventually) decreasing: $a_n \ge a_{n+1}$ 2. $\lim_{n=1}^{\infty} a_n = 0$ Then $\sum_{n=1}^{\infty} (-1)^{n+1}a_n$ converges.	He given

<u>Proof</u> . Suppose {an} is positive, decreasing, and $\lim_{n\to\infty} a_n = 0$. We prove {5 _{2n} } and {5 _{2n+1} } both converge to the same limit. {an} decreasing \Rightarrow aj-aj+1 > 0 Vj. Hence	· ·
For even partial sums: $S_2 = a_1 - a_2 > 0$ $S_4 = (a_1 - a_2) + (a_3 - a_4) = S_2 + (a_3 - a_4) > S_2$ $S_6 = S_4 + (a_5 - a_6) > S_4$ Thus $0 < S_2 < S_4 < < S_{2n} <, so \{S_{2n}\}$ is increasing. Now $S_{2n} = a_1 - (a_2 - a_3) - (a_{4n} - a_{5}) (a_{2n-2} - a_{2n+1}) - a_{2n} \Rightarrow S_{2n} \leq a_1$ Since $\{S_{2n}\}$ is increasing and bounded above, it converges by MCT. Say $a_{2n} \Rightarrow S_{2n} = S$.	For add partial sums. Since Sent = Sent arnt and line an = O. We have home Sent = line (Sent arnt) = StO = S.
So $\lim_{n \to \infty} S_{2n} = S = \lim_{n \to \infty} S_{2n+1}$ so $\lim_{n \to \infty} S_n = S$. Thus the series converges, \blacksquare $S = $	5 * 57 * 56 * 58
$\frac{\mathbf{E}_{n}}{\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}} \text{(Alternating Harmonic Series)} \qquad \qquad$	$\frac{(-1)^{n+1}}{ne^n} = 0, \frac{1}{(ne^n)} < \frac{1}{ne^n} \text{and since } \lim_{n \to \infty} \frac{1}{ne^n} = 0.$
$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \text{ converges by AST (to ln 2)} \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$	(-1) ⁿ⁺¹ converges.
Thm 10: Alternating Series Test (AST) and Estimation of Suppose ∑(-1)" an converges to S. From the proof, the odd/even partial sums approach the actual sum ⇒ S lies between any 2 consective partial sums ⇒ Rn = S-Sn ≤ Sn+1-Sn = ±an+1 =an+1 The error is at most an+1. That is Rn = S-Sn ≤ an+1	Sums and Errors 1 from above/below.
Ex. Find an upper bound on the error if we use S_6 to approximate	e $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^n}$
$ R_{6} \leq a_{7} = \frac{1}{e^{7}} \approx 0.00092 \text{ (rounded up)}$	
Ex.2 How many terms are needed to approximate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!e^n}$ with an error $ R_n \le a_{n+1} = \frac{1}{(n+1)e^{n+1}} \Rightarrow \frac{1}{(n+1)e^{n+1}} \le 0.000005$	of at most 0.000005
n=4: 51.e5 ≈ 0.000056 (nope) n=5: 61.e6 ≈ 0.00000344 (yup) Thus 5 terms are needed.	
If the l st term is positive, then If the l st ter • odd partial sums are overestimates • odd partial • even partial sums are underestimates • even partial	m is negative, then I sums are underestimates I sums are overestimates
E.1 Is S_{4032} an underestimate or overestimate of $\sum_{i=1}^{\infty} \frac{(-n^{n+1})}{ne^n}$? I st term is positive so S_{4032} is an underestimate	· · · · · · · · · · · · · · · · · · ·

5.8 Absolute vs Conditional Convergence
A series Zan is absolutely convergent if Zan is convergent.
$\frac{1}{10} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ is absolutely convergent $\frac{1}{10} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ is not absolutely convergent
since $\sum_{i=1}^{\infty} \left \frac{(-1)^{n+1}}{n} \right = \sum_{i=1}^{\infty} \frac{1}{n}$ since $\sum_{i=1}^{n} \left \frac{(-1)^{n+1}}{n} \right = \sum_{i=1}^{\infty} \frac{1}{n}$
$n_{\overline{z} } = n^{2} n^{2}$ $n_{\overline{z} } = n^{2} n^{2}$ $n_{\overline{z} } = n^{2} n^{2} n^{2}$
Det. Conditionally convergent A series is conditionally convergent it is convergent but not absolutely convergent
I hm II. Absolute Convergence Theorem (ACT) If Σ[a] converges, then Σan is convergent
(but not to the same value unless an ≥ 0 $\forall n$)
Proof
Since $0 = a_n + a_n = a_n + a_n = 2 a_n \Rightarrow a_n + a_n = 2 a_n $
For each n, we have $\sum_{n=1}^{\infty} (a_n + a_n) \leq \sum_{n=1}^{\infty} 2 a_n $
Since $\sum_{n=1}^{\infty} a_n $ converges, so does $\sum_{n=1}^{\infty} 2 a_n $.
$S_0 \sum_{n=1}^{\infty} (a_n + a_n)$ converges by Comparison.
Now $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (a_n + a_n - a_n) = \sum_{n=1}^{\infty} (a_n + a_n) - \sum_{n=1}^{\infty} a_n $
Since both series on the right converges and £. An can be expressed as the difference of 2 convergent series ∴ it also converges ■
This allow us to use tools for anothing some on an endowed by the abodute value
This allows us to use tests to positive series on any series by laring the account value.
$\frac{ S }{ S } \sum_{n=1}^{\infty} \frac{\cos(n) + \sin(n)}{2n^2} = \cos(n + \frac{1}{2}) + \frac{1}{2} +$
We check for convergence by considering $\sum_{k=1}^{\infty} \frac{\cos(k) + \sin(k)}{2}$
$ \cos(h) + \sin(h) \cos(h) + \sin(h) \cos(h) + \sin(h) = 2$
$0 \le \left \frac{1}{2n^2} \right = \frac{1}{2n^2} \le \frac{1}{2$
Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series p=2>1), $\sum_{n=1}^{\infty} \left \frac{\cos(n) + \sin(n)}{2n^2} \right $ converges by Companison
By ACT, $\sum_{n=2}^{\infty} \frac{\cos(n) + \sin(n)}{2n^2}$ converges.
To determine absolute/conditional convergence or divergence.
1. Divergence lest 2. Check for Absolute convergence using Tests for positive series
3. Check for Conditional convergence using AST (if possible)

$\sum_{n=1}^{\infty} \frac{(1) \sqrt{n^{+1}}}{n^{2}}$ converge abs/cond or diverges
$\int_{n=1}^{n} \frac{1}{n^{n+1}} = \lim_{n \to \infty} \frac{1}{n\sqrt{1+\frac{1}{n^{n}}}} = \lim_{n \to \infty} \frac{1}{\sqrt{1+\frac{1}{n^{n}}}} = 0$
n~~ n² **** n² *~~ n ~
·· Div test is incondusive
$\frac{2}{\sqrt{2}} \operatorname{Consider} \sum_{n=1}^{\infty} \left \frac{(-1)}{n^2} \sqrt{n^2 + 1} \right = \sum_{n=1}^{\infty} \frac{\sqrt{n^2 + 1}}{n^2}$
$\lim_{n \to \infty} \frac{\frac{4n^{2}+1}{n^{2}}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{\sqrt{n^{2}+1}}{n} = \lim_{n \to \infty} \frac{n\sqrt{1+\frac{1}{n^{2}}}}{n} = E(0,\infty) $
Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (Harmonic Series), $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n^2 + 1}}{n^2}$ diverges
$\sum \frac{(-1)^{n-1}}{n}$ doesn't converge absolutely
$\frac{1}{3} \text{ We know } \frac{\sqrt{n^2+1}}{2} \ge 0 \text{ Vn and } \frac{\sqrt{n^2+1}}{\sqrt{n^2+1}} = 0 \text{ by } 1.$
Let $f(x) = \frac{\sqrt{x^{-4}1}}{x^2} \Rightarrow f(x) = \frac{\sqrt{x^{-4}2}}{x^2\sqrt{x^2+1}} < 0$ for $x > 0 \Rightarrow \left\{ \frac{\sqrt{n^2+1}}{n^2} \right\}$ is decreasing
· · · · · · · · · · · · · · · · · · ·
$50 \text{ by AS1, } \sum_{n=1}^{\infty} converges$
$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n^2 + 1}}{(-1)^n \sqrt{n^2 + 1}}$ converges conditionally
$n=1$ D^2
The D' Bassanan Thereas
The 12. Rearrangement Theorem $1 + \frac{1}{2}$ a shedded in the second on the terms will still concern to S
Thm 12. Rearrangement Theorem 1. If Σ, an absolutely converges to SCR, then rearranging the terms will still converge to S. 2. If Σ, an conditionally converges to SCR, then rearranging the terms will change the sum to «CR or ±∞.
Thm 12. Rearrangement Theorem 1. If $\overline{\Sigma}$ an absolutely converges to SCR, then rearranging the terms will still converge to S. 2. If $\overline{\Sigma}$ an conditionally converges to SCR, then rearranging the terms will change the sum to \propto CR or $\pm\infty$.
Thm 12. Rearrangement Theorem 1. If \$ an absolutely converges to SER, then rearranging the terms will still converge to S. 2. If \$ an conditionally converges to SER, then rearranging the terms will change the sum to KER or ±∞ 5.9 Ratio Test
Thm 12. Rearrangement Theorem 1. If \$ an absolutely converges to SER, then rearranging the terms will still converge to S. 2. If \$ an conditionally converges to SER, then rearranging the terms will change the sum to <6R or ±∞. 5.9 Ratio Test
Thm 12. Rearrangement Theorem 1. If Ξ an absolutely converges to SER, then rearranging the terms will still converge to S. 2. If Ξ an conditionally converges to SER, then rearranging the terms will change the sum to αER or ±∞. 5.9 Ratio Test Thm 13. Ratio Test
The 12. Rearrangement Theorem 1. If $\frac{1}{2}$ an absolutely converges to SER, then rearranging the terms will still converge to S. 2. If $\frac{1}{2}$ an conditionally converges to SER, then rearranging the terms will change the sum to \propto ER or $\pm\infty$. 5.9 Ratio Test The 13. Ratio Test Let $\frac{1}{2}$ an be a series and assume that $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $LE[0,\infty)$ or $L=\infty$.
Thm 12. Rearrangement Theorem 1. If $\sum_{i=1}^{n} a_i$ absolutely converges to SER, then rearranging the terms will still converge to S. 2. If $\sum_{i=1}^{n} a_i$ conditionally converges to SER, then rearranging the terms will change the sum to \propto ER or $\pm\infty$. 5.9 Ratio Test Thm 13. Ratio Test Let $\sum_{i=1}^{n} a_i$ be a series and assume that $\lim_{i=1}^{n} \frac{a_{min}}{a_m} = L$; $LE[0,\infty)$ or $L=\infty$. 1. If $L=1$, then $\sum_{i=1}^{n} a_i$ converges absolutely.
Thm 12. Rearrangement Theorem 1. If $\frac{5}{2}$ an absolutely converges to SER, then rearranging the terms will still converge to S. 2. If $\frac{5}{2}$ an conditionally converges to SER, then rearranging the terms will change the sum to \propto ER or $\pm\infty$. 5.9 Ratio Test Thm 13. Ratio Test Let $\frac{3}{2}$ an be a series and assume that $\lim_{n \to \infty} \frac{2n+1}{2n} = L$, $LE[0, \infty)$ or $L=\infty$. 1. If $L^{<}1$, then $\frac{3}{2}$ an converges absolutely. 2. If $L^{>}1$ or $L=\infty$, then $\frac{3}{2}$ an diverges. 3. If $L=1$ then the Ratio test is increased.
The 12. Rearrangement Theorem 1. If $\frac{1}{2}$ an absolutely converges to SER, then rearranging the terms will still converge to S. 2. If $\frac{1}{2}$ an conditionally converges to SER, then rearranging the terms will change the sum to \propto ER or $\pm\infty$. 5.9 Ratio Test Let $\frac{1}{2}$ and be a series and assume that $\lim_{n \to \infty} \frac{ a_{n+1} }{ a_n } = L$, $LE[0, \infty)$ or $L=\infty$. 1. If $L=1$, then $\frac{1}{2}$ an converges absolutely. 2. If $L=1$ or $L=\infty$, then $\frac{1}{2}$ an diverges. 3. If $L=1$, then the Ratio test is inconclusive.
Thm 12. Rearrangement Theorem If ∑ an absolutely converges to SER, then rearranging the terms will still converge to S. If ∑ an conditionally converges to SER, then rearranging the terms will change the sum to αER or ±∞. F.9 Ratio Test Thm 13. Ratio Test Let ∑ an be a series and assume that lim and an and an an an and an an
Thm 12. Rearrangement Theorem If \$\sum_{in}\$ an absolutely converges to SER, then rearranging the terms will still converge to S. If \$\sum_{in}\$ an conditionally converges to SER, then rearranging the terms will change the sum to \$\lambda \mathcal{ER}\$ or \$\pm \overline\$. If \$\sum_{in}\$ an conditionally converges to SER, then rearranging the terms will change the sum to \$\lambda \mathcal{ER}\$ or \$\pm \overline\$. If \$\sum_{in}\$ an converges absolutely. If \$\sum_{in}\$ be a series and assume that \$\sum_{in}\$ \$\begin{pmatrix} mathcal{m} mathcal{m} mathcal{m} mathcal{m} mathcal{m} mathcal{m} m} mathcal{m} mathcal{m} mathcal{m} m}\$ If \$\sum_{in}\$ an converges absolutely. If \$\sum_{in}\$ then the Ratio test is inconclusive.
Thm 12. Rearrangement Theorem 1. If \$\vec{2}\$, an absolutely converges to SER, then rearranging the terms will still converge to 5. 2. If \$\vec{2}\$, an conditionally converges to SER, then rearranging the terms will change the sum to <pre></pre> CR or ±∞. 5.9 Ratio Test Let \$\vec{2}\$ and be a series and assume that $ m_n ^{\frac{2n+1}{2n}} =L$, $LE[0,\infty)$ or $L=\infty$. 1. If L<1, then \$\vec{2}\$ an converges absolutely. 2. If L>1 or L=∞, then \$\vec{2}\$ an diverges. 3. If L=1, then the Ratio test is inconclusive. Proof: 1. If 0=L<1, 3rER 3 L <r<1. Since <math> m_n ^{\frac{2n}{2n}} =L<r< math="">, 3NEN 3 Yn≥N, <math> \frac{2n+1}{2n} <r=1< math=""> and <math> <r=1< math="">.</r=1<></math></r=1<></math></r<></math></r<1.
Thm 12. Rearrangement Theorem If ∑, an absolutely converges to SER, then rearranging the terms will still converge to S. If ∑, an conditionally converges to SER, then rearranging the terms will change the sum to CER or ±∞. Factio Test Thm 13. Ratio Test Let ∑, an be a series and assume that the left and and the sum to CER or ±∞. If L=1, then ∑, an converges absolutely. If L=1, then the Ratio test is inconclusive. Proof: If 0=L<1, 3rER 3 L<r<1. Since the laws1 </r<1. Since the laws1 Imm 13. Ratio Test (*)
The 12. Rearrangement Theorem 1. If ∑, an absolutely converges to SER, then rearranging the terms will still converge to S. 2. If ∑, an conditionally converges to SER, then rearranging the terms will change the sum to QER or ±∞. 5.9 Ratio Test Let ∑, an be a series and assume that $\lim_{n \to \infty} \frac{n}{n} = L$, LE[0,∞) or L=∞. 1. If L=1, then ∑, an converges absolutely. 2. If L=1, then ∑, an diverges. 3. If L=1, then the Ratio test is inconclusive. Proof: 1. If 0=L<1, 3rER 3 L <r<1. Since $\lim_{n \to \infty} \frac{n}{n} = L < r$ and $r > 1 = r > 1 = n = 1 < r$ and $r > 1 = r > 1$. Thus $anu < r > 1 = r < 1$, $\sum_{n \to \infty} r > 1 < r > 1 = r > 1 = n = 1 < r > 1 = r < 1 < r > 1 < r <$</r<1.
The 12: Rearrangement Theorem 1. If $\tilde{\Sigma}_{n,\alpha}$ absolutely converges to SER, then rearranging the terms will still converge to 5. 2. If $\tilde{\Sigma}_{n,\alpha}$ an conditionally converges to SER, then rearranging the terms will change the sum to QER or $\pm\infty$. 5.9 Ratio Test Let $\tilde{\Sigma}_{n,\alpha}$ be a series and assume that $\lim_{n\to\infty} \frac{2n\pi i}{n} = L$, $L\in[0,\infty)$ or $L=\infty$. 1. If $L=1$, then $\tilde{\Sigma}_{n,\alpha}$ converges absolutely. 2. If $L=1$ or $L=\infty$, then $\tilde{\Sigma}_{n,\alpha}$ diverges. 3. If $L=1$, then the Ratio test is inconclusive. Proof: 1. If $O=L<1$, $\exists r\in iR \ni L< r<1$. Since $\lim_{n\to\infty} \lim_{n\to\infty} z_{n}r=1 < r \Rightarrow z_{n+1} < r \Rightarrow z_{n+1} < r = z_{n+1} < z_{n+1} < r = z_{n+1} < z_{n+1} < z_{n+1} < r = z_{n+1} < z_{n+1} < z_{n+1} < z_{n+1} < z_{n+1} < r = z_{n+1} < z_{n+1} < $
Thm 12. Rearrangement Theorem If ∑ an absolutely converges to SER, then rearranging the terms will still converge to S. If ∑ an conditionally converges to SER, then rearranging the terms will change the sum to QER or ±∞. If ∑ an conditionally converges absolutely. If L^{<1}, then ∑ an converges absolutely. If L^{<1} or L=∞, then ∑ an converges. If L^{<1} or L=∞, then ∑ an diverges. If L^{<1} or L=∞, then ∑ an diverges. If C[≤]L <
Then 12. Rearrangement Theorem If £ a absolutely converges to SCR, then rearranging the terms will still converge to 5. If £ an conditionally converges to SCR, then rearranging the terms will change the sum to ∝CR or ±∞. If ∑ an be a series and assume that the lameler is a series absolutely. If L=1, then £ an converges absolutely. If L=1, then £ a converges absolutely. If C=1, frefR 3 L=r=1. Since the lameler is inconclusive. Proof: If O=L<1, frefR 3 L=r<1. Since the lameler is inconclusive. Proof: If O=L<1, frefR 3 L=r<1. Since the lameler is inconclusive. Proof: If O=L<1, frefR 3 L=r<1. Since the lameler is inconclusive. Proof: If O=L<1, frefR 3 L=r<1. Since the row is a series and convergent geometric series. By Comparison and using (*), the lameler is a convergent geometric series. By Comparison and using (*), the lameler is a convergent geometric series. By Comparison and using (*), the lameler is a convergent geometric series. By Comparison and using (*), the lameler is a convergent geometric series. By Comparison and using (*), the lameler is a convergent geometric series. By Comparison and using (*), the lameler is a lameler is a lameler is a convergent geometric series. By Comparison and using (*), the lameler is a lameler is a lameler is a lameler is a convergent geometric series. By Comparison and using (*), the lameler is a lamel
Thm 12. Rearrangement Theorem If £ an absolutely converges to SER, then rearranging the terms will still converge to S. If £ an conditionally converges to SER, then rearranging the terms will change the sum to XER or ±∞. 5.9 Ratio Test Thm 13. Ratio Test Let £ an be a series and assume that ¹ / ₁ ^[m]
Thm 12: Rearrangement Theorem If E an absolutely converges to SER, then rearranging the terms will still converge to S. If E an conditionally converges to SER, then rearranging the terms will change the sum to QER or ±∞. 5.9 Ratio Test Thm 13: Ratio Test Let E an be a series and assume that the lambda is a series and assume that the lambda is a series and assume that the lambda is a series. If L=1, then E a converges absolutely. If L=1 or L=∞, then E at is inconclusive. Proof: If O=L<1, BrER B L<1. Since the lambda is inconclusive. Proof: If O=L<1, BrER B L<1. Since the lambda is inconclusive. Proof: If O=L<1, BrER B L<1. Since the lambda is inconclusive. Proof: If O=L<1, BrER B L<1. Since the lambda is a convergent geometric series. By Comparison and using (9), E and converges. Why? E and I amather and the series assolutely. If L=1 or L=∞, then BNEN B Vn=N, amather and +ralant+ralant+ralant+ralant+ralant Thus E an converges absolutely. If L=1 or L=∞, then BNEN B Vn=N, amather and the lambda is a series. By Comparison and using (9), E and converges. Why? E and I amather and the MEN B Vn=N, amather and +ralant+ralant+ralant+ralant Thus E an converges absolutely. If L=1 or L=∞, then BNEN B Vn=N, amather and the lambda is a convergent geometric series. So the series diverges by Div Test. So the series diverges by Div Test.
 Thm 12. Rearrangement Theorem 1. If \$\subset{s}_{nan}\$ absolutely converges to \$\subset{S}\$, then rearranging the terms will still converge to \$\subset{S}\$. 2. If \$\subset{s}_{nan}\$ conditionally converges to \$\subset{S}\$\$, then rearranging the terms will change the sum to \$\partial{C}\$\$ or \$\pm \pi \no\$\$. 5.9 Ratio Test Let \$\subset{s}_{nan}\$ be a series and assume that \$\subset{m}_{m} = 0\$ and \$\subset{m}_{m} = 1\$. Let \$\subset{s}_{nan}\$ be a series and assume that \$\subset{m}_{m} = 0\$ and \$\subset{m}_{m} = 1\$. Let \$\subset{s}_{nan}\$ be a series and assume that \$\subset{m}_{m} = 0\$ and \$\subset{m}_{m} = 1\$. Let \$\subset{s}_{nan}\$ be a series and assume that \$\subset{m}_{m} = 0\$ and \$\subset{m}_{m} = 1\$. Let \$\subset{s}_{nan}\$ be a series and assume that \$\subset{m}_{m} = 0\$ and \$\subset{m}_{m} = 1\$. Let \$\subset{s}_{nan}\$ be a series and assume that \$\subset{m}_{m} = 0\$ and \$\subset{m}_{m} = 1\$. Let \$\subset{s}_{nan}\$ be a series and assume that \$\subset{m}_{m} = 0\$ and \$\subset{m}_{m} = 1\$. Let \$\subset{s}_{nan}\$ be a series and assume that \$\subset{m}_{m} = 0\$ and \$\subset{s}_{nan}\$ be a series. 3. If \$L = 1\$, then the Ratio test is inconclusive. Proof: If \$P = L < 1\$, \$\subset{s}_{nan}\$ diverges. If \$P = 1\$, \$\subset{s}_{nan}\$ be a convergent geometric series. By Comparison and using \$\left(0\$, \$\subset{s}_{m}\$ and \$\subset{s}_{nan}\$ for here \$\subset{s}_{m}\$ and \$\subset{s}_{m}\$ a
 Then 12. Rearrangement Theorem If \$\frac{1}{2}\$, an absolutely converges to SER, then rearranging the terms will still converge to 5. If \$\frac{1}{2}\$, an conditionally converges to SER, then rearranging the terms will change the sum to QER or ±∞. 5.9 Ratio Test Let \$\frac{1}{2}\$ an be a series and assume that \$\frac{1}{2}\$ \$\

Ex. $\left \sum_{n=1}^{\infty} \frac{5^n}{n!}\right $ Using Ratio test, $\left \lim_{n \to \infty} \left \frac{5^{n+1}}{(n+1)!} \cdot \frac{n!}{5^n} \right = \lim_{n \to \infty} \frac{5}{n+1} = 0 < $ $\therefore \frac{95}{n+1} \frac{5^n}{n!}$ converges absolutely.	$5.2 \underset{n=1}{\overset{c}{\leftarrow}} \underbrace{(-1)^{n+1} n 2^{n}}_{5^{n}}$ Using Ratio test, $\lim_{n \to \infty} \left \frac{(-1)^{n+2} (n+1) 2^{n+1}}{5^{n+1}} \cdot \frac{(-1)^{n+1} (n+1)}{(-1)^{n+1}} \right $ $\therefore \underbrace{\sum_{n=1}^{\infty} (-1)^{n+1} (n 2^{n})}_{5^{n}} \text{converge}$	<u>5</u> ⁿ -1) ⁿ⁺¹ n2 ⁿ = him (-1)2(n+1 5n es absolutely.	$\frac{1}{2} = \frac{2}{5} \lim_{n \to \infty} \frac{n+1}{n} = \frac{2}{5} < 1.$	· · · · · · · · · · · ·
Ex.3 $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ Using Ratio test, $\lim_{n \to \infty} \left \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n}{n^n} \right = \lim_{n \to \infty} \frac{(n+1)^{n+1}}{(n+1)!n^n} = \lim_{n \to \infty} \frac{\sum_{n=1}^{\infty} \frac{n^n}{(n+1)!}}{(n+1)!}$	$\prod_{gen} \left(\frac{n+1}{n}\right)^n = \lim_{n \to \infty} \left(\left + \frac{1}{n}\right ^n = e > \right $	<4 <u>∞ n³+4</u> N=1 n ⁵ -1 Using Ratio test, lim (n+1) ³ +4 · n ⁵ -1 (n+1) ⁵ -1 · n ³ +4 = ∴ inconclusive → try	$= \lim_{n \to \infty} \left(\frac{(n+1)^{3} + 4}{n^{3} + 4} + \frac{n^{3} - 1}{(n+1)^{5} - 1} \right) = + = 1$	
5.10 Root Test Let ∑ian be a series and assu 1. If L<1, then ∑ian converg 2. If L>1 or L=∞, then ∑ian 3. If L=1, then the Root test	me limen[lan]=L, LE[0,∞) es absolutely. In diverges. is inconclusive.	or L= ∞.	· ·	· · · ·
$E_{k} \left[\sum_{n=1}^{\infty} \left(\frac{\ln(n+1)}{n^{3}} \right)^{n} \\ \text{Using Root test,} \\ \lim_{n \to \infty} \left[\left(\frac{\ln(n+1)}{n^{3}} \right)^{n} \right]^{\frac{1}{n}} = \lim_{n \to \infty} \frac{\ln(n+1)}{n^{3}} \frac{L_{k}^{2}}{n^{3}} \\ \therefore \text{ series converges}$	$\lim_{n \to 00} \frac{\frac{1}{n+1}}{3n^2} = \lim_{n \to 00} \frac{1}{3n^2(n+1)} = 0$			· · · · · · · · · · · · · · · · · · ·
Note. If the Root test (L=1),	he Ratio test will fail. ns. x >1 x >1			· · · ·
Thm 14. Polynomial vs Factoria For xER, the st. = 0.	al Growth			· · · · · · · · · · · · · · · · · · ·
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$ \sum_{n=1}^{\infty} \frac{n!}{n!} \text{ diverges.} $ 5.10 Root Test $ \begin{array}{c} \text{Thm 15. Root Test} \\ \text{Let } \sum_{n=1}^{\infty} n \text{ be a series and assult in the Let 1, then } \sum_{n=1}^{\infty} n \text{ convergent } \\ \text{Let } \sum_{n=1}^{\infty} n \text{ be a series and assult } \\ \text{If } L < 1, \text{ then } \sum_{n=1}^{\infty} n \text{ convergent } \\ \text{2. If } L > 1 \text{ or } L = \infty, \text{ then } \sum_{n=1}^{\infty} n \text{ convergent } \\ \text{3. If } L = 1, \text{ then the Root test, } \\ \lim_{n \to \infty} \left[\left(\frac{\ln(n+1)}{n^3} \right)^n \right]^{\frac{1}{n}} = \lim_{n \to \infty} \frac{\ln(n+1)}{n^3} \frac{L^2}{n^3} \\ \therefore \text{ series converges} \\ \text{Note. If the Root test } (L=1), \text{ the Root test } (L=1), \text{ the Root test } (L=1), \text{ the Root test } (\ln n)^n \ll n^n \ll n^n, \\ \frac{1}{n^n} \ll \frac{1}{n!} \ll \frac{1}{n!} \ll \frac{1}{n!} \ll \frac{1}{n!} \ll \frac{1}{(\ln n)^n} \\ \text{Then 14. Polynomial vs Factorial For section, we get n = 0. \end{cases} $	me $\lim_{n\to\infty} a_n = L, L \in [0, \infty)$ es absolutely. In diverges. Is inconclusive. $ a_n = \lim_{n\to\infty} \frac{1}{3n^n(n+1)} = 0$ the Ratio test will fail. x > 1 al Growth	or L= ∞.		

Recap of Series Test

Absolute Convergence or Divergence.	Conditional Convergence
 Sums of Geometric and Telescoping Series 	· AST (alternating series)
\mapsto try to spot these series	→ look for (-1)" or (-1)" '
→ if it asks to find the sum, it's likely one of these	
 Divergence Test (any series) 	
→ try this 1 st , unless there's a factorial	
· Integral Test (positive series)	
→ last resort when all else fails	
is don't forget continuous positive, decreasing	
· Comparison Test (opsitive series)	
hast assort when all else fails	
· I (T (miting spring)	
the series of the form "powers of n"	
Schest annabie and a	
La davit franct 1=0 and =00 and man any disated	
• Rote To at (an annual)	
Inchio lest lang series/	
Tactorials	
→ almost" geometric series	
→ L=1 gives no into	
· Koot lest (any series)	
⇒ when all terms have a power ot n	